Price Space Perpetuals

A New Primitive for Leveraged Derivative Markets

Ambient Finance

Abstract

Perpetual futures today are the dominant instrument for leveraged crypto trading, but they are structured around a set of time-based mechanisms whose economic effects are difficult to reason about, especially under stressed market conditions. This paper introduces two related primitives that define an alternative framework for leveraged linear exposure: 1) the price race digital, a first-touch contract that resolves when spot reaches one of two price boundaries; and 2) the price space perpetual, a perpetual-like derivative without any dependency on wall clock time. In an idealized setting, price race digitals admit a simple state-based valuation and can be composed to synthesize ongoing linear exposure equivalent to a price space perpetual. This renewal perspective yields a different interpretation of key perpetual-market mechanisms: funding appears as a stochastic roll cost; leverage as the number of adverse price intervals a bankroll can survive; liquidation as price space bankroll exhaustion; and ADL as failed renewal under counterparty scarcity.

1. Introduction

Perpetual futures are the dominant instrument for leveraged trading in crypto markets. Their appeal is easy to understand. They offer linear exposure, leverage, and no fixed maturity. But the economic logic of the product is less simple than the user-facing surface. A live perpetual is not just "spot with leverage." It is an ongoing directional contract coupled with a stack of adjustment mechanisms: periodic funding, margining, liquidation thresholds, backstop layers, and auto-deleveraging. In normal conditions that stack is mostly invisible. In stressed conditions it can become unstable and unpredictable.

This paper explores an alternative architecture. Can perpetual-like directional exposure exist without any dependency on wall clock time? We introduce a new binary derivative, the **price race digital**, that resolves when a reference price first reaches one of two boundaries. From repeated renewal of price races as the price traverses a ladder of brackets, we then build a new type of perpetual in **price space** (instead of time space).

Many of the economic functions traders associate with perpetuals may be expressible through a renewal system of bounded local claims. And doing so yields a different and in some respects cleaner interpretation of carry, leverage, liquidation, and deleveraging.

1.1 Motivation

The motivating friction is in how today's perp users are asked to think about directional exposure through several disjoint layers at once. A trader who holds a standard perpetual is implicitly interacting with at least the following:

- A continuously quoted mark or tradable perp price
- A funding rule intended to anchor that price to spot over time
- Margin formulas that determine how much adverse movement can be absorbed
- Liquidation mechanisms that force reduction when those formulas fail
- Venue-level buffers such as backstop liquidity, socialized loss, or ADL

The resulting product can feel economically fragmented. Funding charges for imbalance through time. Liquidation determines whether exposure can survive marked losses. ADL and backstops determine what happens when liquidation itself is insufficient. The user experiences one instrument, but the economic adjustment is distributed across disparate control systems. The same trader may ask:

- Is the perp rich or cheap to spot?
- Is funding worth paying or receiving?
- How far am I from liquidation?
- How much of this market's apparent depth is real under stress?
- What happens if the other side cannot be closed cleanly?

The price space framing begins from the possibility that these are not inevitable features of perpetual-like exposure. They may instead be features of one particular architecture: a **time space instrument** whose balancing logic acts through clock time.

The question is whether exposure can instead be built from bounded local contracts whose continuation is keyed to movement through price levels rather than to the passage of time. If that is possible, then funding, continuation, deleveraging, and exit may all become questions asked at explicit renewal boundaries in price space rather than at partially separate layers distributed through wall-clock time.

1.2 Properties of an Ideal Perpetual

A useful way to frame the problem is to first ask what traders actually want from a perp-like instrument.

The first and most basic property is **linear directional exposure**. They want exposure whose first-order behavior is legible and does not drift simply because time passes or volatility changes. The point is to avoid carrying unnecessary gamma, theta, vega, or path-driven complexity in what is meant to function as a directional instrument.

The second property is **leverage**. Without leverage, many of the practical reasons traders use perpetuals disappear. Relative to holding spot, a perp-like product should allow meaningful directional sensitivity per unit of committed capital.

The third property is **indefinite maintainability**. A product can be directional and levered yet still fail if it forces the user to manage hard expiry. What traders value in a perpetual is not immortality for its own sake, but the ability to maintain exposure without worrying about the complexity of expiries and rolls.

The fourth property is **no reliance on shortfall-bearing backstops**. This is the hardest requirement and the one conventional perps compromise most. If a contract creates open-ended liabilities, then some external mechanism must eventually absorb or reassign those losses: liquidation vaults, insurance funds, ADL, socialized loss, or other mechanisms. A cleaner architecture would avoid embedding that open-ended shortfall risk in the primitive itself.

A fifth, softer property is **legible and modest carry**. Traders prefer continuation costs that are understandable, relatively stable in normal conditions, and not wildly disconnected from the economic function they are serving.

1.3 Time Space vs Price Space

The cleanest way to understand the proposal is through an architectural contrast.

A conventional perpetual can be summarized as follows:

- One standing linear directional contract
- A spot or index reference process
- Funding rules to anchor the contract through time
- Margining and liquidation to control open-ended risk
- Venue-level backstop layers when liquidation is insufficient

In that system, entry and exit are priced in the order book, while continuation of stressed imbalance is governed through a mixture of funding, liquidation, insurance, and ADL. The economic logic is distributed across several time-based mechanisms.

A price space perpetual looks different:

- One active local price race on the current bracket
- A renewal rule that determines the next bracket after boundary touch

- Renewal markets that price continuation into the next local state
- Bankroll rather than debt-style margin as the natural state variable
- Non-renewal, renewable liquidity, and renewal skew as the key quantities governing survival and stress

In this system, the product is a rule for repeatedly reconstituting local exposure as price moves through adjacent intervals. Entry into the current local state and continuation into the next state are related, but not identical. Standard perps are **time space instruments** whose balancing mechanisms operate through clock time. Price space perps are **renewal systems in price space** whose balancing mechanisms operate through fixed boundaries in the underlying price.

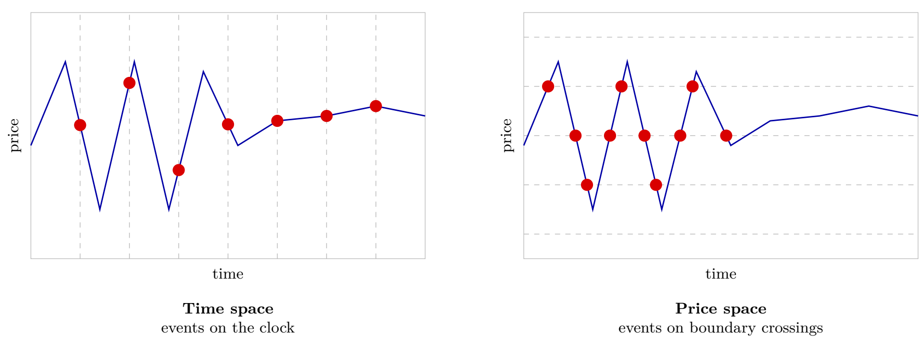


Figure 1: Same price path, two event clocks. On the left, time space meters events on a vertical grid of clock ticks: every interval yields an event regardless of how much price has moved. On the right, price space meters events on a horizontal grid of boundary crossings: events cluster when price is active and vanish when it is quiet. In a standard perpetual, funding accrues on the left grid; in a price space perpetual, renewals accrue on the right.

1.4 Contributions

The remainder of the paper proceeds as follows. Section 2 places the proposal against the background of perpetual futures and binary directional markets. Section 3 defines the price race primitive and establishes its local first-exit pricing properties. Section 4 shows why, in continuous price space, bounded perp-like exposure naturally leads to renewal across adjacent brackets. Section 5 develops the market architecture of a price space perpetual, including renewal pricing, local basis, and funding as stochastic roll cost. Section 6 reinterprets leverage, liquidation, and ADL in bankroll and renewal terms.

2. Background

2.1 Perpetual Futures

Perpetual futures were not originally introduced merely as a crypto-native convenience product. The deeper idea is to build a derivative with no fixed maturity that still remains tied to an underlying spot price. Because a perpetual has no expiry date, it cannot rely on ordinary convergence at maturity. Some other mechanism must keep the contract economically anchored.

Modern perpetuals solve that anchoring problem through an explicit time-based control system. A live perpetual is a standing linear exposure plus a collection of adjustment mechanisms: basis monitoring, periodic funding, margin requirements, liquidation rules, insurance or backstop layers, and eventually auto-deleveraging and emergency closeout rules. These pieces are familiar enough to practitioners that they can appear natural, but they are better understood as one point in a wider design space for maintaining non-expiring directional exposure.

The academic and empirical literature has sharpened three points that matter for this paper.

First, funding is not an incidental fee. It is part of the anchoring mechanism that keeps a maturity-free contract tied, however imperfectly, to a reference market. Whether one describes the perp through no-arbitrage bounds, random-maturity analogies, or explicit feedback rules, the recurring theme is the same: without an anchoring mechanism, a perpetual can drift away from the economic object traders think they are holding.

Second, basis in perpetual markets is inseparable from market structure. In practice, deviations between perp price and spot are shaped not only by ideal arbitrage logic, but also by capital constraints, fragmentation, execution costs, inventory risk, and the willingness of traders to warehouse one-sided exposure. This is why funding and basis cannot be understood purely as cosmetic overlays. They are where crowding, risk appetite, and market quality become visible.

Third, leverage turns those basis and liquidity frictions into non-linear stress events. Classical work on funding-liquidity spirals and newer crypto evidence point in the same direction: once a levered position is maintained through margin rather than full collateralization, local dislocations can force deleveraging before long-run convergence has any chance to assert itself. Liquidation, insurance, and ADL are therefore not peripheral annoyances. They are the mechanism by which the system decides who absorbs losses when a system exceeds its collateral limits.

It is helpful to describe a conventional perpetual as a **time space instrument**. The trader holds one continuously marked directional contract, and the balancing mechanisms act through clock time. Funding accrues on a schedule, marks update continuously, and deleveraging occurs when time-evolving prices and margin constraints force intervention. The alternative explored in this paper is not "a better funding formula." It is a different geometry for the exposure itself.

2.2 Up/Down Binary Markets

The second adjacent family is the world of binary and up/down directional contracts. At the highest level, these markets partition an event into mutually exclusive outcomes and attach prices to them. In prediction-market language, they are winner-take-all contracts. In derivative language, they sit near digital, one-touch, and barrier-style payoffs. In retail and crypto settings, they often appear as very short-horizon bets on whether an asset will finish above or below a threshold over some specified interval.

On the positive side, bounded binary claims are simple, fully collateralizable, and often easy to aggregate across traders. They can reveal directional beliefs cleanly, and modern market-design work shows that even thin binary markets can support information aggregation when the trading mechanism is designed well. This is one reason binary-like contracts remain conceptually attractive: they package directional uncertainty into a compact and legible payoff.

They are also attractive as speculative vehicles in their own right. Products such as very short-horizon up/down markets, 0DTE options, and other capped-loss high-sensitivity contracts appeal because they offer a familiar combination: explicit maximum loss, fast resolution, strong local directional sensitivity per dollar committed, and no open-ended liability for the holder. That combination is not the same as long-run investment utility, but it is real product demand.

There is a constructive lesson in the barrier and digital-options literature. Finance is already familiar with contracts that embed stopping rules, knock-ins, knock-outs, and threshold-triggered state changes. The idea of making a price level operational inside the contract is not alien to modern finance.

3. Price Race Digitals

We start by defining the local derivative primitive from which a price space perpetual is built. We describe a new primitive, the **price race digital**, a bounded first-touch directional claim between two price levels. It matters for the broader construction because inside a bracket its price can still be read as a local directional state. That combination of bounded payoff and local directional sensitivity makes it a useful building block for an eventual perpetual architecture.

3.1 Formal Definition

Let S_t denote the reference price of an underlying asset, as determined by a specified oracle or reference market process, and let t_0 denote the inception time of a new contract. Fix two barriers

$$L < U,$$

and suppose the reference price at inception lies strictly between them:

$$S_{t_0} \in (L, U).$$

The basic contract is a first-touch claim on which barrier is reached first. In the idealized continuous formulation, the first exit time from the interval is

$$\tau := \inf\{t \geq t_0 : S_t \leq L \text{ or } S_t \geq U\}.$$

The **price race digital** consists of two complementary claims:

- an **Up claim**, which pays one unit if the upper barrier is reached first;
- a **Down claim**, which pays one unit if the lower barrier is reached first.

Their terminal payoffs are

$$X^{\uparrow} = \mathbf{1}_{\{S_\tau \geq U\}}, \qquad X^{\downarrow} = \mathbf{1}_{\{S_\tau \leq L\}}.$$

Under a well-defined resolution rule, exactly one claim pays one and the other pays zero, so

$$X^{\uparrow} + X^{\downarrow} = 1.$$

This complementarity matters for two reasons. First, it makes the primitive analytically simple: the market is partitioning one first-passage event into two mutually exclusive directional outcomes. Second, it makes the claim naturally interpretable as a local directional instrument rather than as a free-standing bet detached from price discovery.

As a practical product, the contract must be parameterized not only by the barriers but also by the reference and settlement rules. It is therefore useful to write the primitive as

$$(L, U, t_0, \mathcal{O}, \mathcal{R}),$$

where $\mathcal{O}$ specifies the oracle or reference-price process and $\mathcal{R}$ specifies the resolution semantics: what counts as a touch, how observations are timestamped, how stale data or jumps are treated, and how ties or edge cases are resolved. In that sense, the price race is not merely an abstract first-passage claim. It is a first-passage claim under explicit market-design semantics. Section 7 returns to these implementation choices in detail.

The point is that the primitive is both local and bounded. It is local because it only concerns which edge of a bracket is reached first. It is bounded because the payoff is binary and fully collateralizable. Those two features distinguish it from a standard linear perpetual while still leaving room for meaningful directional behavior inside the interval.

3.2 Symmetric Brackets and Implied Mark

Most of the construction in this paper is easiest to express in a symmetric bracket. Let the bracket center be K and the half-width be $h > 0$, so that

$$L = K - h, \qquad U = K + h.$$

The associated first exit time becomes

$$\tau := \inf\{t \geq t_0 : S_t \notin (K - h, K + h)\}.$$

The Up and Down payoffs can then be written as

$$X^{\uparrow}_{K,h} = \mathbf{1}_{\{S_\tau \geq K+h\}}, \qquad X^{\downarrow}_{K,h} = \mathbf{1}_{\{S_\tau \leq K-h\}}.$$

Let $p \in [0, 1]$ denote the market price of the Up claim. Ignoring fees and other frictions, the Down claim then has price $1-p$. A quote near $p = 1/2$ corresponds to a balanced bracket, while prices above or below $1/2$ indicate that the market assigns greater weight to one side of the bracket being reached first.

That binary quote can be mapped back into a local linear state. Define the **implied mark**

$$S_{\text{impl}} := K + h(2p - 1).$$

Equivalently,

$$p = \frac{S_{\text{impl}} - (K - h)}{2h}.$$

This expression linearly embeds the binary quote into the bracket interval. Several basic interpretations follow immediately:

- if $p = 1/2$, then $S_{\text{impl}} = K$;
- if $p > 1/2$, then the implied mark lies above the center;
- if $p < 1/2$, then the implied mark lies below the center;
- as $p \to 1$, the implied mark approaches the upper boundary;
- as $p \to 0$, the implied mark approaches the lower boundary.

The implied mark is not a separate traded instrument. It is a linear-equivalent state variable extracted from the price race quote. Its purpose is interpretive. It transforms a bounded binary contract market into the language traders use for directional products: mark, implied price within the bracket, and local basis relative to spot. Later sections use exactly this mapping to connect price race prices to perp-style notions of quoted mark and basis.

This symmetric parameterization also clarifies what the primitive is and is not doing. The contract does not say that the underlying is itself linear inside the interval. Rather, it says that the market's price for the first-touch claim can be represented as a local position within that interval. That is enough to make the instrument useful as a building block for a larger directional system.

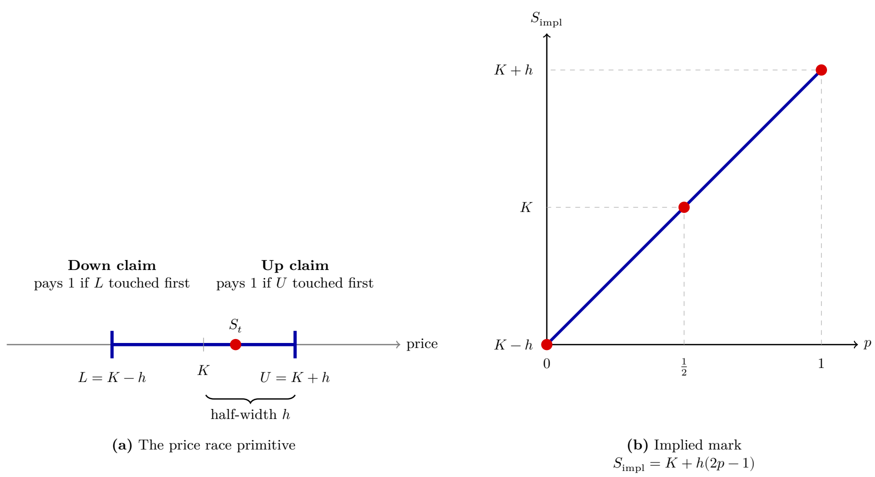


Figure 2: **(a)** A price race is a first-touch claim between a lower barrier $L = K - h$ and an upper barrier $U = K + h$. The Up claim pays 1 if the upper barrier is reached first; the Down claim pays 1 if the lower barrier is reached first. **(b)** The Up-claim market price $p \in [0, 1]$ embeds linearly into an implied mark inside the bracket. $p = 0$ maps to the lower barrier, $p = 1/2$ to the center, and $p = 1$ to the upper barrier.

3.3 Valuation Intuition

The price race digital has a discrete terminal payoff, but its market value moves continuously. That is the source of its directional usefulness. The instrument is binary at settlement, yet locally and continuously state-sensitive until settlement.

The simplest way to see this is through the implied-mark mapping above. Since

$$p = \frac{S_{\text{impl}} - (K - h)}{2h},$$

the local sensitivity of the Up claim price to the implied mark is

$$\frac{\partial p}{\partial S_{\mathrm{impl}}} = \frac{1}{2h}.$$

This gives a natural local delta interpretation. One unit of Up-claim notional contributes approximately $1/(2h)$ units of directional sensitivity with respect to the implied mark. The narrower the bracket, the larger that local sensitivity becomes. Conversely, wider brackets supply less local sensitivity per unit notional.

This scaling is the key intuition:

- **narrow brackets** produce higher local delta and more linear-like exposure per unit notional;
- **wide brackets** produce lower local delta and coarser directional exposure.

Suppose a trader wants Q units of local spot-equivalent directional exposure inside the bracket. A natural first-order sizing rule is to hold

$$n = 2hQ$$

units of the Up claim, since then the local delta of the position is approximately

$$\frac{\partial (np)}{\partial S_{\mathrm{impl}}} = n \cdot \frac{1}{2h} = Q.$$

This does not mean the price race is globally identical to a linear contract. The contract still resolves into a bounded binary payoff at the boundary. But inside the bracket, and over sufficiently local movements, a properly sized position in the binary can approximate the first-order behavior of a linear directional instrument.

That approximation becomes especially intuitive near a symmetric bracket whose quote is close to $1/2$. Around that state, small upward moves raise the Up claim price and small downward moves lower it in a nearly affine way, with slope determined by the bracket width. This makes the price race behave less like a fixed opinion on a far-off event and more like a compact directional slice of price space.

The same intuition also identifies the core tradeoff that recurs throughout the paper. Making h small sharpens directional sensitivity and reduces the capital needed to synthesize a given local delta. But it also increases renewal frequency and makes the system more sensitive to jump risk, oracle discreteness, and boundary overshoot. The primitive is therefore flexible, but not free of geometry: its leverage-like character is directly controlled by bracket width.

3.4 Derivative Properties of the Price Race Digital

The price race digital combines three properties that do not usually sit together:

1. its payoff is bounded
2. its value is locally directional
3. its directional sensitivity can be tuned through bracket width

First, the contract is **bounded-payoff**. A long Up claim can lose at most the premium paid and gain at most the unit payoff minus that premium. There is no open-ended liability and no need to support the position through mark-to-market losses larger than committed capital. In that sense, the primitive is fully collateralizable by construction.

Second, the contract is **locally affine** in economic behavior even though it is not globally linear in payoff. Before resolution, the market price responds directionally to changes in the perceived location of the underlying within the bracket. Through the implied-mark mapping, that response is approximately linear over local interior moves. This is the precise sense in which a bounded binary can still support delta-like exposure.

Third, the contract is **geometrically tunable**. The half-width h governs how much local directional sensitivity is obtained per unit notional. Decreasing h increases sensitivity, making the primitive more linear-like over shorter price intervals. Increasing h decreases sensitivity, making the primitive coarser and more resilient to microstructure noise. The primitive does not come with one fixed leverage profile. Its leverage intensity is a direct consequence of bracket design.

This combination is what makes the price race a compelling local building block for a perpetual architecture. A trader can obtain strong first-order directional exposure from a bounded contract without taking on the shortfall risk of an undercollateralized linear book. Said differently, the primitive can deliver meaningful delta per unit of committed capital while keeping loss bounded to the capital actually deployed in the claim.

That does not remove economic risk. A trader can still overexpose themselves by sizing too aggressively, renewing too often, or choosing brackets that are too narrow for the underlying process. But the mode of risk is different. The primitive itself is not fragile, because it hides no open-ended liability. It becomes fragile only when assembled into an overly ambitious renewal strategy.

The price race digital is best understood as a bounded local directional primitive. This combination of properties is what makes the next step possible: using repeated price race positions to synthesize a perp-like exposure.

4. Perps as Renewals

What happens when we hold not just a single price race to expiry, but repeatedly re-enter the next price race? In this section we explore **renewals** — atomic entries into the next price race at the exact expiry of the previous — as a way to translate the piecewise linear exposure of individual price races into global linear exposure.

4.1 The Impossibility of a Single Bounded Globally Linear Perpetual

Begin with the strongest possible version of the desired exposure. Suppose one wants a single contract on an unbounded price domain whose value is globally affine in spot:

$$V(s) = a + bs, \qquad b \neq 0,$$

for all relevant spot levels s. This is the purest representation of linear directional exposure: constant delta, no curvature, and no drift in the pricing rule simply because time passes or local volatility changes.

But such a contract cannot also be bounded. If $b > 0$, then $V(s) \to +\infty$ as $s \to +\infty$; if $b < 0$, then $V(s) \to -\infty$ as $s \to +\infty$. On any unbounded price domain, nonzero global linearity and bounded payoff are incompatible. The same point holds if the domain is unbounded in either direction. A nontrivial globally linear claim necessarily creates unbounded future obligations for one side of the contract.

That observation has direct market-structure implications. If a product offers globally linear non-expiring exposure, then some balance sheet in the system must be willing to absorb potentially unbounded losses. In traditional perps that burden is distributed across counterparties, margin rules, liquidation processes, insurance funds, and (in extreme cases) ADL or socialized-loss. These mechanisms are how standard perps can preserve linearity, leverage, and non-expiry. The product works by allowing the contract-level obligation to remain open-ended, then building external machinery around it to insulate from failure modes.

This suggests an impossibility result. No single claim can satisfy all four of the following properties simultaneously:

- Nontrivial linear directional exposure
- Leverage
- Indefinite maintainability
- No shortfall-bearing backstop

4.2 Localizing Exposure Without Shortfall

If global linearity is impossible under bounded payoff, the only remaining path is to localize exposure. Instead of asking for a single contract that behaves linearly over the whole price domain, we use a sequence of "building block" contracts that behave linearly over a bounded region of price space. That building block is the price race digital developed in Section 3.

Inside a bracket $[L, U]$, the price of the digital can be read as a local affine state whose directional sensitivity is controlled by bracket width. In a local sense it preserves the perp-like exposure that traders care about.

This localization is a necessary response to the above incompatibility. If shortfall risk is excluded at the contract level, then exposure must be housed in claims whose obligations are themselves bounded. That is why the relevant design problem is not "how do we build a safer standard perp?" but rather "what bounded local claim can temporarily substitute for a linear one inside a controlled region of price space?"

Since no single everlasting bounded linear claim exists, the solution must instead be a succession of local claims, each valid within its own price neighborhood. This is why the product must be spatial rather than temporal. Profit and loss is defined in price space, and therefore the tradable product must be as well.

4.3 Renewal by First Exit

Localizing exposure solves the boundedness problem but introduces a new one: non-expiry. What happens if a trader wants indefinite maintainability?

In the continuous-price idealization, the natural answer is renewal at first exit. Suppose the current local claim is defined on a bracket $[L, U]$. As long as spot remains in that interval, the claim retains the local exposure properties for which it was selected. Once price exits the interval, those properties no longer hold. The local neighborhood has been exhausted. To preserve exposure, the system must immediately instantiate a new local claim on the next relevant neighborhood.

The correct renewal time is therefore

$$\tau := \inf\{t \geq t_0 : S_t \notin (L, U)\}.$$

Rolling earlier would reintroduce an unnecessary time clock into the construction, breaking the price space exposure. Rolling later is impossible in the idealized local design, because after exit the claim loses its bounded local approximation. First exit is not merely one possible stopping rule among many. In the continuous setting it is the canonical stopping rule for preserving indefinite localized exposure.

Continuity of the price path matters here. With continuous motion, the process exits when the spot price is *exactly* equal to the boundary price. That gives the system a clean renewal event and a natural adjacent successor region. The idealized model has no need to adjudicate skipped brackets or overshoot semantics. (These dynamics are explored in Section 7.) For the continuous model, first exit supplies the exact event structure needed for clean renewal.

This is the key bridge from local bounded exposure to perpetual-like exposure. Non-expiry is no longer derived from one immortal contract. It is synthesized by repeatedly renewing exhausted local claims at the exact price and time the boundary is reached.

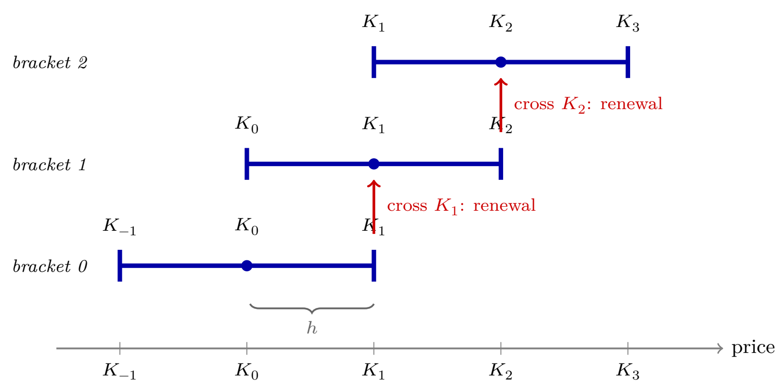


Figure 3: Renewal stitching across overlapping brackets. Each bracket has half-width h, and grid centers K_j are spaced by h, so each bracket's boundaries coincide with the centers of its two neighbors. When price exits the active bracket through a boundary, the next active bracket is centered on that crossed boundary. Repeated upper-boundary crossings walk the active bracket along the grid to the right, as shown; lower-boundary crossings walk it to the left. Indefinite directional exposure is synthesized by renewal without any single contract ever being unbounded.

4.4 Corollary: Necessity of a Renewal System

Conditional on organizing the product in continuous price space without tolerating unbounded shortfall risk, the architecture is essentially determined:

1. Global exposure must be decomposed into bounded local claims
2. Local claims must renew when their bracket is exited
3. The canonical local claim is the price race digital

That is the conceptual payoff of the argument. Price space perpetuals are not introduced because they are novel or elegant. They arise naturally if we insist on preserving linearity, leverage, non-expiry, and boundedness simultaneously. A renewal system of price races is the only architecture consistent with all four.

5. From Price Race to Price Space

In a mechanical sense, how do we actually construct perps from price race digitals?

This section's central move is to show how a sequence of price races can be assembled into a coherent perp-like venue through renewal pricing in continuation markets.

The mechanism can be understood at two levels. At the derivative level, repeated price races create an event-rolled directional system whose local economics resemble a perpetual. At the market-structure level, renewal markets, persistent order translation, and liquidity coordination make that system tradeable as a single coherent market rather than as a loose collection of disconnected binaries.

5.1 Multi-Period Price Races

Fix a bracket half-width $h > 0$ and an ordered grid of possible centers spaced by h:

$$K_j = K_0 + hj, \qquad j \in \mathbb{Z}.$$

Bracket j is then

$$[K_j - h, K_j + h],$$

so adjacent brackets overlap by h on each side, and each boundary of bracket j coincides with the center of an adjacent bracket.

Suppose at time t_0 the active bracket is centered at K_0 and the active contract is the corresponding price race. Let τ_1 be the first time price exits that bracket. The next active bracket is centered on the boundary just crossed: if price exits through the upper boundary at $K_0 + h$, the new bracket is centered at $K_1 = K_0 + h$; if price exits through the lower boundary at $K_0 - h$, the new bracket is centered at $K_{-1} = K_0 - h$. Iterating this procedure produces a sequence of renewal times

$$\tau_1 < \tau_2 < \tau_3 < \cdots$$

and a sequence of active brackets indexed by the realized path through the ladder.

At each renewal, the trader can either stop or continue by entering the next local price race. A maintained long (short) delta position is therefore not a standing exposure on one contract. It is a policy of repeatedly selecting the Up (Down) oriented directional leg on the next active bracket. The position survives not because a single derivative never expires, but because the market keeps reconstituting the next local exposure when the previous one resolves.

The local delta calculation from Section 3 makes the connection precise. In a symmetric bracket of half-width h, one unit of the Up claim has local delta approximately $1/(2h)$. A trader seeking Q units of local spot-equivalent exposure therefore holds approximately

$$n = 2hQ$$

contracts in each fresh bracket. Each completed bracket excursion then generates PnL of approximately

$$\Delta\Pi = Q(\pm h),$$

with the sign determined by whether the directional move favored or opposed the trader. If the position is renewed immediately after each resolution, then the trader experiences a sequence of local directional gains and losses tied to successive price traversals of length h. The resulting object is a directional system whose “intervals” are first-passage events in price space rather than increments of clock time.

This event-rolled structure is the basic sense in which repeated price races generate perpetual exposure. A standard perpetual keeps a trader exposed continuously while price and mark evolve through time. A price space perpetual keeps the trader exposed piecewise through a succession of local renewal events. Exposure continuity is not primitive, but synthesized from repeated local continuation.

There is another important economic point hiding inside that construction. The local price race does not need to be justified only as backend plumbing for a synthetic perp. It may also be a directly attractive product for speculators who want short-horizon, bounded-loss directional exposure. In that sense, the same bracket market can serve at least three kinds of demand at once:

- traders renewing a perp-like position through price space
- traders who want to trade the local race explicitly as a standalone contract
- market makers and arbitrageurs warehousing or hedging local first-touch risk

That overlap matters because it suggests that renewal demand and standalone race demand can share liquidity rather than fragmenting into unrelated venues.

5.2 Worked Path Example

It is useful to make the renewal logic concrete with a simple pathwise example. Suppose the current bracket is

$$[95, 105]$$

with center $K_0 = 100$ and half-width $h = 5$. Consider three traders:

- **Renewing long**, who buys the Up claim and intends to keep renewing long-delta exposure
- **Renewing short**, who buys the Down claim and intends to keep renewing short-delta exposure
- **Standalone race trader**, who buys the current Up claim but has no intention of renewing after settlement

Suppose price first touches 105. Then the active Up claim resolves successfully and the Down claim expires worthless. At that moment, the standalone race trader is simply done. The trader entered one bounded local contract, the event happened, and the position is over. By contrast, the renewing long treats that touch as a continuation boundary rather than as an endpoint. The next active bracket is centered on the crossed boundary, $K_1 = 105$, and so becomes

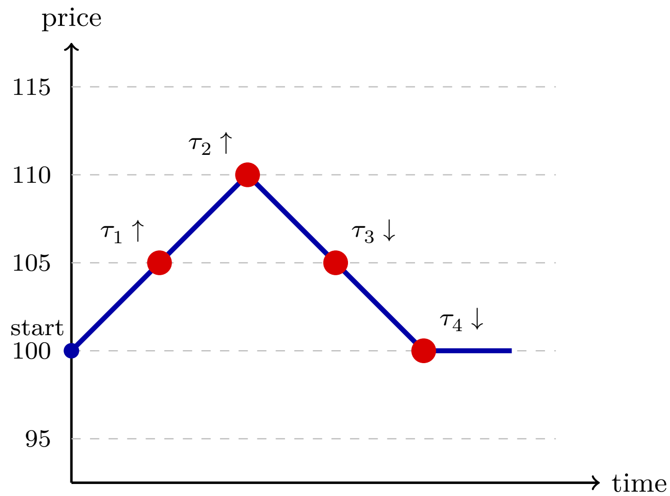
$$[100, 110].$$

The long can acquire the next Up claim to remain exposed. The renewing short can either buy the Down claim in the new bracket to remain short-delta, or stop.

Now suppose price fails to continue upward and instead reverses, touching 100 before reaching 110. The second bracket then resolves downward. The renewing long loses on the renewed leg, the renewing short gains on the renewed leg, and the standalone trader who exited after the first bracket bears no exposure to the reversal at all.

This small example illustrates three points at once:

First, a price space perpetual is not one everlasting contract. It is a policy of repeatedly buying the next local claim after each resolution. Second, non-renewal is a real structural action. A trader can simply step off at any boundary and exit. Third, the same local bracket market can serve both renewal demand and standalone race demand. The trader who wants one fast bounded directional shot and the trader who wants a continuing perp-like position are trading the same instrument, skinned differently.



bracket P&L:	$\tau_1 \uparrow$	$\tau_2 \uparrow$	$\tau_3 \downarrow$	$\tau_4 \downarrow$
renewing long	$+hQ$	$+hQ$	$-hQ$	$-hQ$
renewing short	$-hQ$	$-hQ$	$+hQ$	$+hQ$
stops after τ_1	$+hQ$	—	—	—

Figure 4: A worked renewal path. Starting from bracket $[95, 105]$, price touches 105 at τ_1, recentering the active bracket on the crossed boundary to give $[100, 110]$. It continues up through τ_2 at 110 (new bracket $[105, 115]$), reverses through τ_3 at 105 (new bracket $[100, 110]$) and τ_4 at 100 (new bracket $[95, 105]$). The table shows bracket-level P&L in the centered, no-skew benchmark: the renewing long wins the up-touches and loses the down-touches; the short experiences the mirror; a trader who stops after τ_1 pockets the initial win and is not exposed to later brackets.

5.3 Trade at Renewal Markets

With renewal as the core transition, a natural market follows: the market for the next active race before it goes live. We call this a **trade-at-renewal market**. It is analogous to a trade-at-settlement (TAS) market. Participants can submit orders for the next active price race digital before the current market resolves.

In the idealized symmetric centered case, a fresh bracket that opens exactly around the current reference state has a natural benchmark value. If the next bracket is centered on the renewal state, then the fresh Up claim begins at the symmetric benchmark

$$p^\star = \frac{1}{2}.$$

That benchmark does not imply the market must clear at 1/2. It means that 1/2 is the no-skew reference point for a newly centered race. A trade-at-renewal market then allows the actual opening price to be discovered before the moment of transition. Participants can pre-commit to buy or sell continuation at whatever price reflects their willingness to warehouse the next unit of directional risk.

This market performs several functions at once.

First, it gives continuing traders a place to secure the next unit of exposure without waiting for the previous bracket to resolve. Second, it gives opposing traders and market makers a place to reveal how much capital they are willing to commit to the next renewal. Third, it turns renewal from a purely mechanical event into an actual market-clearing event.

This point matters for the broader architecture because renewals are the market in which the system prices open interest. The active price race digital is the market for local exposure. The renewal market is the market for continuation.

In a standard perpetual, the ordinary order book prices entry and exit, while continued imbalance is only partially priced through funding. Liquidation rules, insurance, ADL, and open-interest operate separately to control how much stressed exposure can actually remain in the system. In a renewal-based architecture, those questions intersect directly at the boundary. The market asks:

- Who wants the next unit of exposure?
- Who is willing to warehouse the other side?
- How much size can continue?
- And at what price?

5.4 Price Race Skew as Basis

Once a renewal price can differ from its symmetric benchmark, the system acquires a local notion of basis. Let p_n denote the opening price of the Up claim in the n-th renewed bracket, with center K_n and half-width h. The corresponding implied mark is

$$S_{\text{impl},n} = K_n + h(2p_n - 1).$$

If the renewal state is exactly centered, the actual reference price at the start of the new bracket is K_n. In that idealized case, the local basis is simply

$$B_n := S_{\text{impl},n} - K_n = h(2p_n - 1).$$

Thus:

- if $p_n > 1/2$, the implied mark is rich to the centered benchmark and long-delta continuation is expensive;
- if $p_n < 1/2$, the implied mark is cheap to the centered benchmark and short-delta continuation is expensive.

More generally, if the renewal state is not exactly centered, one compares $S_{\mathrm{impl},n}$ to the actual renewal reference price S_{τ_n}:

$$B_n := S_{\mathrm{impl},n} - S_{\tau_n}.$$

Renewal skew in binary-price space maps directly into a local basis in implied-mark space. This is a close analogue to the perp-versus-spot relation in standard perpetuals. A standard perp can trade rich or cheap to spot. Here, the active price race can imply a mark rich or cheap to the idealized renewal price.

5.5 Funding as Stochastic Roll Cost

With renewal skew as a local basis, the equivalent of "funding" for a price space perp is the skew paid or received over successive renewals in a single direction. If one side of the market repeatedly pays above the symmetric benchmark to continue exposure, then that side is paying carry through repeated renewal premia. Funding is no longer best described as a transfer per unit time. It is better described as a **stochastic roll cost**.

To see this, consider a trader maintaining approximately Q units of spot-equivalent exposure using $n = 2hQ$ contracts per bracket. Let

$$\epsilon_i := p_i - \frac{1}{2}$$

denote the renewal skew paid by the long-delta side in bracket i, with the sign reversed when short-delta continuation is the crowded side. If N_T is the number of completed renewals by calendar time T, then the cumulative continuation cost paid by the crowded side is approximately

$$\mathrm{RollCost}_T \approx 2hQ \sum_{i=1}^{N_T} \epsilon_i.$$

The crucial feature is that N_T is endogenous. It depends on how often price traverses the bracket spacing. In quiet markets, few renewal events occur and little carry is paid over a fixed calendar interval. In active markets, many renewal events occur and the same per-renewal skew can accumulate quickly. The effective clock-time funding burden is therefore driven by realized price-path activity:

$$\frac{\mathrm{RollCost}_T}{T} \approx 2hQ \cdot \frac{1}{T} \sum_{i=1}^{N_T} \epsilon_i.$$

This is the sense in which funding accrues in event time or volatility time rather than in pure clock time. Standard perps charge for maintaining crowded directional imbalance through time. Price space perps charge for renewing crowded directional imbalance through realized movement in price space.

That said, the analogy to funding is imperfect. In standard perps, funding is often an explicit periodic cash transfer engineered to pull a quoted perp mark toward spot. It is therefore a mechanism for charging already-open imbalance through time. But it is not, by itself, the full market for whether that imbalance remains open. Liquidation, ADL, insurance, and open-interest caps remain partially separate mechanisms governing how much stress the system can actually carry.

In the renewal system however, carry is more endogenous and directly tied to continuation itself. The trader pays it by repeatedly entering at skewed opening prices relative to the local benchmark. In that sense, renewal is not *just* analogous to funding. It is a direct market for selling the next unit of open interest. One can summarize the contrast this way:

- Standard-perp funding rents open interest through time
- Renewal pricing sells the next unit of open interest at the boundary

That is why renewal markets can unify more of the economic logic of continuation than ordinary funding does. They do not merely charge for imbalance after positions are open. They price whether the next unit of imbalance will exist at all, in what size, and on what terms.

The distinction changes the character of hedged basis trading. Standard perp basis arbitrage typically warehouses time-based carry together with funding uncertainty, basis convergence risk, and liquidation plumbing. Price space basis arbitrage instead warehouses a sequence of local, event-based, finite price races as defined and repeated "bets" with legible edges. The trade is less "hold this spread and earn funding until it converges" and more like "repeatedly decide whether X skew justifies warehousing Y units of price race risk."

5.6 Perpetual as Renewal System

The preceding subsections suggest a high-level definition.

A **price space perpetual** is a market architecture specified by:

- A bracket geometry, typically a ladder of adjacent intervals in price space
- A local derivative primitive, namely the price race on the active bracket
- A renewal rule determining which bracket becomes active after each boundary hit

- A market-clearing mechanism for the next local race, including trade-at-renewal functionality
- A continuation policy describing how a trader maintains, modifies, or terminates exposure across successive renewals

Under this definition, the product is not one everlasting claim. It is a rule for maintaining directional exposure through repeated local renewal. Continuation is the central state transition. The trader remains in the product only by successfully acquiring the next unit of local exposure when the previous one ends. This framing clarifies the architecture in several ways.

First, it explains why the system can be perpetual-like without being perpetual in the literal single-contract sense. The persistence lies in the renewal rule, not in contractual immortality. Second, it clarifies where basis and funding live: in successive renewal prices. Third, it prepares the later reinterpretation of leverage and liquidation. If continued exposure depends on repeatedly funding the next renewal, then bankroll and continuation become the natural objects of risk analysis.

The same definition also makes room for an important dual use of the primitive. The local price race can be traded as an explicit standalone contract, while also serving as the continuation object for a synthetic perp. A practical venue therefore need not choose between “binary product” and “perp market.” It can let the two uses share the same local books through two different lenses.

6. Leverage, Liquidation, and Market Impact

How should risk, leverage and liquidation be interpreted in the framework of a price space perpetual?

6.1 Leverage as Bankroll

Consider a trader targeting approximately Q units of spot-equivalent directional exposure through a ladder of symmetric price races with half-width h. From Section 3, the local delta of one Up claim is approximately $1/(2h)$, so the trader holds roughly

$$n = 2hQ$$

claims per bracket. If the opening price of the relevant leg in bracket i is p_i, then the capital required to fund that renewal is

$$C_i = np_i = 2hQp_i.$$

In the centered no-skew benchmark $p_i = 1/2$, this becomes

$$C_i = hQ.$$

That quantity is the basic capital unit of the strategy. It is the bankroll consumed to maintain Q units of local directional exposure over one bracket. The narrower the bracket, the smaller the capital required to maintain the same target exposure. This is the basic source of leverage in the price space architecture.

Let B_i denote the trader's bankroll immediately before entering bracket i. In an idealized fixed-width system, one may therefore think of the trader's leverage-like capacity not primarily as notional divided by collateral, but as the ratio

$$\frac{B_i}{hQ},$$

which measures how many benchmark renewal units of the target exposure the trader can fund. Equivalently, it measures how many adverse local excursions of size h the trader can survive before continuation fails.

This way of thinking is closer to the economic reality of the product than the usual margin-language inherited from standard perps. In a standard perpetual, a trader often experiences leverage as a mark-price distance to liquidation generated by a maintenance-margin formula. In a renewal-based product, the trader can instead think in terms of bankroll survival:

- How much capital is available for the next renewal?
- How large a local exposure each renewal consumes?
- How many adverse intervals can be absorbed before the strategy stops?

That interpretation is one of the most intuitive advantages of the price space framing. The system does not pretend leverage is anything other than the finite capacity to keep paying for exposure maintenance.

6.2 Liquidation as Bankroll Exhaustion

Once leverage is expressed through bankroll, liquidation also changes meaning. It should no longer be described as the venue seizing a marked position and force-selling it. It is better described as the first renewal at which the trader can no longer fund the next continuation.

To formalize that idea, let $\xi_i \in \{+1, -1\}$ denote the directional outcome of bracket i, measured from the perspective of the trader's position. In the centered benchmark, the trader's bracket-level PnL is approximately

$$\Delta\Pi_i \approx Q\xi_i h.$$

If the trader also pays renewal skew $\epsilon_i := p_i - 1/2$ when continuation is crowded, then a convenient reduced-form bankroll recursion is

$$B_{i+1} \approx B_i + Q\xi_i h - 2hQ\epsilon_i.$$

The first term is the incoming bankroll. The second is directional PnL from the bracket excursion. The third is the continuation premium paid to renew exposure at a skewed opening price.

Liquidation is then the stopping time

$$\ell := \inf\{i : B_i < C_i^\star\},$$

where $C_i^\star$ is the capital required to fund the trader's intended next renewal at the desired size and current renewal price. In the simplest fixed-size centered case, $C_i^\star \approx hQ$. With renewal skew, adaptive bracket changes, or partial continuation rules, $C_i^\star$ can vary by state.

This framing has a major conceptual consequence. The insolvent account does not need to be liquidated through a distressed trade in order for the loss to be realized. The loss is already crystallized by the completed bracket sequence. What fails is not settlement of the previous bracket. What fails is the ability to buy the next one. Liquidation is therefore a natural and pre-determined absorbing boundary in account state rather than a separate emergency act of execution.

This is a cleaner and more defensible risk story. If the reference process is trustworthy, then the relevant liquidation event is simply the renewal boundary at which continuation is no longer affordable. The trader can understand exactly where exposure lasted, why bankroll was insufficient for the next continuation, and therefore why the strategy stopped. That is a much more transparent object than a forced liquidation sale triggered by a separate mark-price and margining subsystem.

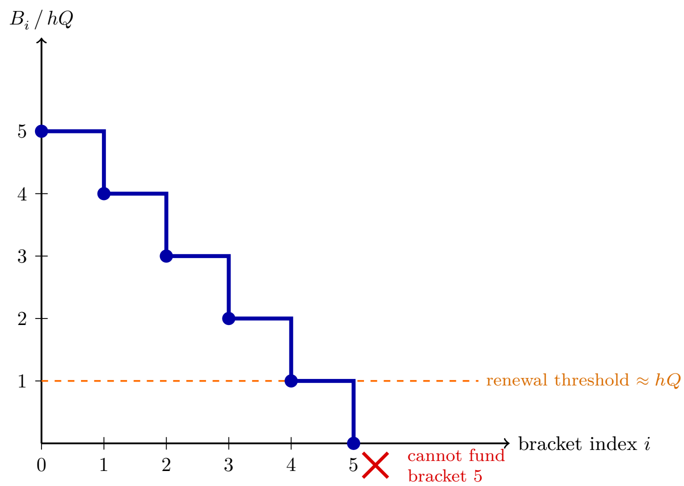


Figure 5: Bankroll evolution under a sequence of adverse brackets (centered, no-skew case). At each renewal, bankroll falls by hQ. Continuation requires $B_i \geq hQ$ to buy the next unit of exposure. The trader survives five adverse brackets starting from $B_0 = 5hQ$; when $B_5 = 0$ falls below the renewal threshold, bracket 5 cannot open and the position fails to continue. Liquidation is absorption at this boundary, not forced sale.

6.3 Bracket Sizing as Leverage Constraint

Bracket width is the main geometric control on effective leverage. This follows directly from the relations above.

First, local directional sensitivity per claim is

$$\Delta_{\text{claim}} \approx \frac{1}{2h}.$$

Second, the benchmark capital required to fund Q units of exposure over one bracket is

$$C \approx hQ.$$

So shrinking h does two things at once. It raises delta per unit claim notional, and it lowers the capital needed to fund a given target exposure for one local renewal.

This is why narrow brackets feel levered. The trader can acquire a given local exposure with less bankroll committed to each renewal. Conversely, wider brackets reduce sensitivity and require more capital to maintain the same target delta, thereby lowering effective leverage.

There is, however, an important nuance. In the continuous idealization, a trader with bankroll B and target exposure Q can survive approximately

$$m \approx \frac{B}{hQ}$$

adverse bracket outcomes, each of size h. The total adverse spot distance that can be absorbed before exhaustion is therefore approximately

$$mh \approx \frac{B}{Q},$$

which is independent of h in the frictionless continuous benchmark. Narrow brackets do not magically create more continuous-model loss capacity. What they do is repartition that capacity into smaller, more frequent renewal checkpoints.

That distinction is central. Bracket width is not merely a leverage dial in the static sense. It is simultaneously:

- A delta dial
- A capital-per-renewal dial
- A renewal-frequency dial
- A stress-fragility dial under discrete observation and gaps

Narrow brackets increase effective leverage and force the system to re-clear exposure more often in price space. This is beneficial because it gives the market more frequent opportunities to detect fragility and stop continuation. But it also makes the system more sensitive to skipped-path effects, overshoot, and jump compression. Wide brackets do the opposite: lower leverage, slower renewal cadence, and more tolerance of discontinuity.

This is why bracket sizing is the true leverage constraint in the architecture. Traditional perps adjust leverage primarily by changing margin parameters on top of a fixed contract. Price space perpetuals adjust leverage directly through the geometry of the local claim itself.

6.4 Liquidation Pressure as Renewal Imbalance

Liquidation costs do not disappear in the renewal architecture. They are relocated.

In a traditional perpetual, once a trader is liquidated, the venue typically forces the account's position either into the open market or into a liquidation backstop mechanism. That concentrates several burdens on the already-failing account and its immediate counterparties:

- Forced execution
- Slippage against a stressed order book

- Liquidation penalties and fees
- Spillover into insurance funds, ADL, or socialized loss if the exit cannot clear cleanly

In a price space perpetual, the insolvent account instead stops at the renewal boundary. The account is not required to dump exposure through an additional distressed trade to realize its loss. The market-clearing consequences of its disappearance reappear in a different place: shifting the equilibrium of the next renewal continuation.

That means the economic problem becomes one of market-impact allocation. Who bears the cost of the removed exposure? The answer is that the cost shifts from distressed execution by the failing account *to* collective repricing among the surviving positions.

This is a meaningful redistribution. The weakest participant is no longer necessarily the one forced to cross the market at the worst possible time. Instead, the remaining market absorbs the stress passively through wider renewal skew, smaller renewable liquidity, or both.

That is not costless for the system, but it is often healthier. The already-underwater account is no longer additionally penalized by market impact from a forced liquidation trade. The burden is borne through renewal repricing by participants who still have capital and can choose whether to continue under the new equilibrium terms. In that sense, the architecture turns liquidation from an execution problem into a price-discovery problem.

6.5 Renewals as a Market for Position Transfer

Renewals are not merely entry venues for the next active bracket. They are the mechanism through which exposure is transferred from traders who cannot or will not continue to traders who are willing to warehouse the next local risk.

This can be understood as a handoff market. Suppose one side of the book has suffered losses and part of that side cannot renew. The next renewal market then asks a concrete question: who is willing to continue this exposure, at what size and at what price?

That is precisely the question a liquidation engine is trying to answer in standard perps. But here it is answered through a renewal market rather than administratively forced execution.

This also suggests a more granular model of continuation preferences. Each trader can be thought of as having a renewal-price tolerance or renewal-skew budget. Some may continue at mild skew, some only at extreme compensation, and some not at all. As imbalance rises, the renewal price moves and traders self-select into continuation or exit. The transfer of exposure happens through price-mediated willingness rather than out-of-band liquidation logic.

It also suggests a different role for specialist outside capital. In standard perps, basis traders, market makers, and liquidation backstops are often separate businesses. In a renewal-based market, those roles can overlap. A desk that sees renewal skew as rich relative to spot and fair continuation value may step in not only as an arbitrageur, but as the capital that allows stressed exposure to find a counterparty. The same actor is simultaneously trading basis and supplying continuation liquidity.

That overlap is important because it makes the decision cleaner. The outside capital provider is not necessarily underwriting an open-ended convergence trade with uncertain future funding and liquidation path. It is looking at a single bounded renewal state with a visible local skew, an explicit size decision, and a fresh chance to reassess at the next boundary. That does not eliminate risk, but it moves backstop participation to a tractable framework of explicit bankroll sizing on repeated bets rather than warehousing an indefinitely stressed spread.

This is one of the deeper architectural differences between the two systems. Standard perps distribute funding, margining, liquidation, and ADL across partially separate subsystems. A renewal-based price space market compresses more of that adjustment into one clearing question at the boundary: what exposure can renew, and at what price?

That is why renewals should be viewed not only as a timing convention, but as a market for position transfer.

6.6 ADL as Counterparty Scarcity

Auto-deleveraging is the limiting case of that logic. In standard perps, ADL is an opaque emergency subsystem that interrupts profitable positions because liquidation plus insurance capacity was insufficient to absorb stressed counterparty losses. Economically, however, the underlying problem is simple: there is not enough opposing balance sheet to take the other side cleanly.

In a price space perpetual, that same problem becomes easier to interpret. ADL-like stress is the visible consequence of counterparty scarcity at renewal. One side has run out of bankroll or refuses continuation, and the market cannot find enough replacement capital within any acceptable range of renewal terms. The result is not a mysterious extra layer on top of the product. It is the failure of the renewal market to clear the full desired continuation size.

This perspective does not eliminate ADL in substance. In sufficiently extreme stress, the renewal market may still fail to clear enough size, wherein a forced reduction of open positions may be required. But the path to that limit can be much more market-mediated.

One reason that path may be healthier is that renewal skew can be a directly visible inducement to outside capital. If stressed continuation clears at unusually rich terms, a specialist trader or vault can evaluate that opportunity as a bounded local bet with explicit bankroll at risk, rather than as a vague promise of future funding on an open-ended position. In that limited but important sense, renewal markets may mobilize a wider pool of backstop capital by being cleaner and more accessible.

One can think of the stress progression as follows:

1) Under mild imbalance, renewal skew widens modestly
2) Under moderate imbalance, some traders voluntarily decline renewal because skew exceeds their tolerance
3) Under severe imbalance, renewable size shrinks sharply and only highly compensated counterparties remain
4) Under total imbalance, the market cannot clear *any* continuation, and all positions close at the boundary price

This is a more legible progression than the usual jump from normal trading to opaque queue-based deleveraging. Price adjustment happens first. Then quantity reduction emerges through non- and partial renewal. ADL becomes counterparty scarcity made visible through renewal price and renewable size. The user can understand: replacement capital demanded wider terms than I accepted, therefore continuation failed.

That is a much more natural story than "the venue's emergency queue selected me." It is also potentially better for interface design. Instead of surfacing opaque ADL rankings, a renewal-based venue could surface quantities users can actually reason about: renewal skew, renewable fraction, continuation probability, and visible depth of opposing bankroll.

The broader conclusion of this section is that leverage, liquidation, and deleveraging all fall out naturally from the renewal-based architecture. Leverage is the ability to keep funding local exposure. Liquidation is bankroll exhaustion at a renewal boundary. Market impact is allocated through renewal repricing. ADL is the extreme form of counterparty scarcity in the renewal market.

7. Further Work

In this paper we explored a new financial instrument, the price space perpetual, within the idealized case of continuous price discovery on the underlying spot. This idealization allowed us to derive tractable properties and develop a precise architecture around the behavior of the instrument across a variety of normal and stressed conditions. However the continuity of the observed price process is not a practical reality in real-world markets. Discretization occurs for two reasons: oracle sampling frequency on the one hand, and discontinuities and microstructure noise in the spot price itself on the other.

In later papers, we will explore how the price space perpetual behaves under discretization from both an empirical and theoretical perspective. We will suggest practical mitigations to preserve most of the ideal properties of the continuous case. We will also demonstrate a process for selecting bracket sizing, and show how continuity can be treated as an asymptote of discretization.

In addition to discretization, later papers will explore practical market structure design for the price space perpetual and optimal ways to unify the parallel price discovery in the active price race and next renewal market. We'll explore unique properties of the price space perpetual under manipulation and cornering conditions. Finally we'll explore the economics, term structure and market structure of the renewal market and how it's likely to price against time-based funding.